VIBRATION ORDER SHIFT AND ITS COMPUTATION

M. Turgay Bengisu, General Motors Corporation, Michigan.

A frequency shift in the vibration spectrum of rotating shafts and discs is observed when such vibrations are transmitted to non-rotating machine components. The same frequency shifts are also seen when vibrations of a rotating body is measured from a non-rotating coordinate system. The shift in the vibration frequencies is due to a coordinate transformation from the rotating to non-rotating coordinate system and the amount of frequency shift is equal to the rotational frequency of the rotating body.

Measuring the vibrations of rotating shafts and discs is frequently a necessary first step in understanding their dynamic behavior. Such vibration measurements often involve employing non-contact displacement or velocity transducers placed in a non-rotating coordinate system. The dynamic behavior of a rotating body may also be inferred from the measurement of vibrations transmitted to a non rotating machine component. In either case, the frequencies of the measured vibrations are shifted by an amount equal to the rotation frequency of the shaft (or disc) [1-5] and the frequency shifts could occur towards higher and/or lower frequencies.

The simplest form of this interesting phenomenon is the behavior of an unbalanced shaft. Consider a rotating shaft, statically bent due to an imbalance force on it. An observer (i.e. a strain gauge) fixed on the shaft will measure static strain (zero frequency) since the shaft is bent but not vibrating. However, when the shaft is observed from a non-rotating coordinate system (i.e. its bending displacement is measured with a non-contact transducer), it will appear as vibrating transversely at a frequency equal to the rotation frequency of the shaft. Likewise, any vibrations transmitted through the bearings of the shaft will be at the frequency of shaft rotation. Hence, the vibration frequency shifts upward from zero (i.e. static bending) in the rotating coordinate system to the frequency of rotation when observed from a non-rotating coordinate system.

In this simple case, it is easy to see how the vibration frequencies are observed to be different in different coordinate systems. In general, however, frequency shifts may be significantly more complicated and some computation is usually needed to transform the measured spectra. The remainder of this article is devoted to formulating the frequency shift phenomenon that occurs when vibrations of rotating shafts (and discs) are measured from non-rotating coordinate systems. The more complicated subject of transmitted vibrations will not be covered here other than to say that the formulation given is also applicable to transmitted vibrations, with some modification, when the bearing forces are direct consequence of vibrations of the rotating body.

Bending Vibrations

Many rotating bodies such as shafts exhibit bending vibrations as a result of the forces acting on them. The bending deflection is in radial direction from the undeformed center of the shaft and it can be measured with non-contact transducers as depicted in Figure 1. Figure 1 shows only the cross section of the shaft in the plane of measurement. The circular cross section of the shaft is assumed to be preserved as the shaft bends and approaches to the transducers. If the displacement transducers are calibrated so that the distances measured during the undeformed state of the shaft produce zero outputs, any deviations from this initial state will be a measure of bending of the shaft.

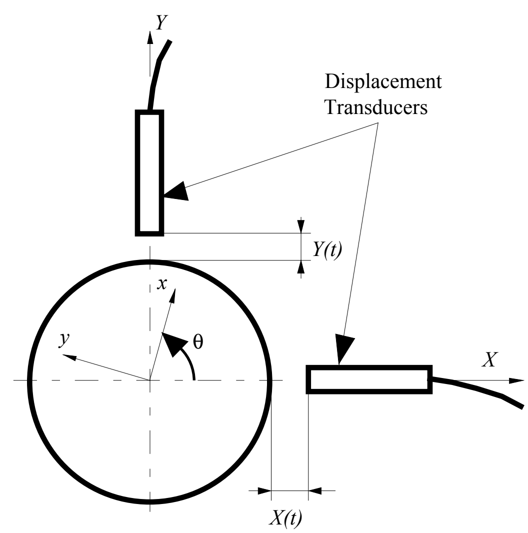


Figure 1. Positioning of non-contact displacement transducers to measure the bending vibrations of a rotating shaft.

The x-y coordinate system shown in Figure 1 rotates with the shaft and its position is measured by the angle θ. The problem in question is to describe the measured deflections $X(t)$ and $Y(t)$ in the x-y coordinate system.

The coordinate systems of Figure 1 are also shown in Figure 2 in more detail. The thin solid circle in the figure depicts the undeformed position of the shaft in the plane of measurement. The coordinate system of the basis vectors **I**, **J**, **K** is non-rotating (i.e. stationary), while the coordinate system with the basis vectors **i**, **j**, **k** is rotating at the same angular velocity as the shaft. The unit vectors **K** and **k** are collinear and the position of the basis vectors **i**, and **j**, is measured by the angle θ.

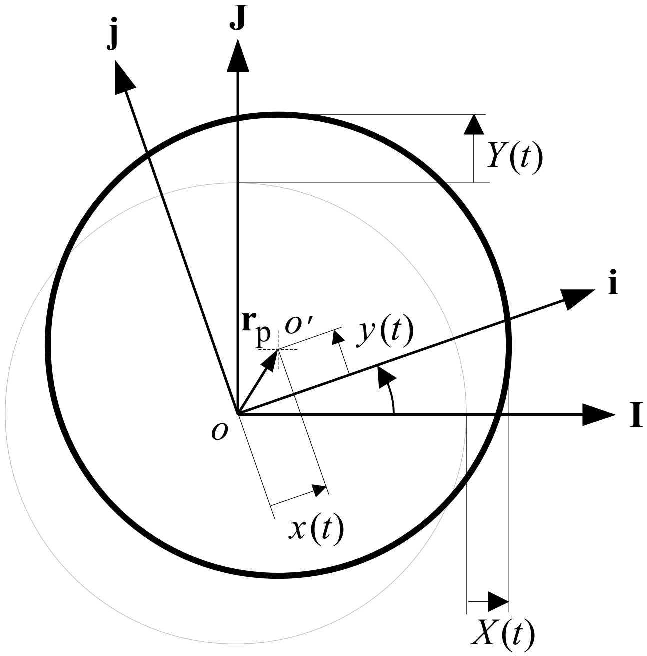


Figure 2. Definition of the non-rotating **I**, **J**, **K** and the rotating **i**, **j**, **k** basis.

Also shown in Figure 2 is the deformed position of the shaft (thick solid circle) with center of shaft displaced from point *o* to *o′*. The position vector $\mathbf{r}_\mathrm{p}$ extends from the origin *o* of coordinates to the center of the deflected shaft *o′* and can be written as

$$\mathbf{r}_\mathrm{p} = x(t)\mathbf{i} + y(t)\mathbf{j}. \tag{1}$$

In equation (1), the components of vibratory displacement, $x(t)$ and $y(t)$ are with respect to the rotating coordinate system. The same position vector can also be expressed with respect to a stationary coordinate system **I**, **J**, **K** by substituting the coordinate transformation

$$\begin{aligned}\mathbf{i} &= \mathrm{Cos}\,\theta\,\mathbf{I} + \mathrm{Sin}\,\theta\,\mathbf{J}\\ \mathbf{j} &= -\mathrm{Sin}\,\theta\,\mathbf{I} + \mathrm{Cos}\,\theta\,\mathbf{J}\end{aligned} \tag{2}$$

in equation (1). Hence, the center of the shaft in the stationary coordinate system, is defined as

$$\mathbf{r}_\mathrm{p} = \left(x(t)\mathrm{Cos}\theta - y(t)\mathrm{Sin}\theta\right)\mathbf{I} + \left(x(t)\mathrm{Sin}\theta + y(t)\mathrm{Cos}\theta\right)\mathbf{J} \tag{3}$$

The components of $\mathbf{r}_\mathrm{p}$ in directions **I** and **J** in equation (3) are the displacements *X*(*t*) and *Y*(*t*) of the rotating shaft with respect to the stationary coordinate system. That is,

$$\begin{aligned}X(t) &= x(t)\mathrm{Cos}\,\theta - y(t)\mathrm{Sin}\,\theta\\ Y(t) &= x(t)\mathrm{Sin}\,\theta + y(t)\mathrm{Cos}\,\theta\end{aligned} \tag{4}$$

Displacement transducers placed in a stationary coordinate system would measure *X*(*t*) and *Y*(*t*). Solving equations (4) for $x(t)$ and $y(t)$ yields the displacements that would have been measured in the rotating coordinate system.

$$\begin{aligned}x(t) &= X(t)\mathrm{Cos}\theta + Y(t)\mathrm{Sin}\theta\\ y(t) &= -X(t)\mathrm{Sin}\theta + Y(t)\mathrm{Cos}\theta\end{aligned} \tag{5}$$

The angular position θ of the shaft with respect to the stationary basis is a function of time. If the rotation speed is constant, θ is given by $\theta = \omega_\mathrm{o} t$, where ω_o is the angular velocity and $\dot{\theta} = \omega_\mathrm{o} = \mathrm{Constant}$. However, it is not necessary that the rotation speed be constant to employ equations (5), since, $\theta(t)$ can be calculated from $\theta(t) = \int \dot{\theta}(t)dt$ if the angular velocity is measured.

Vibrations of Rigid Discs

Wobbling vibrations of discs, when measured from stationary coordinate system have the same frequency shift characteristics. It is difficult to measure such wobbling vibration directly on a body-fixed coordinate system of a rotating disc. However, if the disc is rigid, the displacement of its periphery, in the co-axial direction of its axis of rotation, can be measured by non-contact transducers installed in a stationary coordinate system. This section describes how such vibrations, measured from a stationary coordinate system, can be transformed to the rotating coordinates.

Figure 3 depicts a rotating and vibrating disc with three coordinate systems defined at the center of its rotation. The coordinate system with the basis vectors **I**, **J**, **K** is non-rotating but fixed at the center of the disc. The basis vectors **i**, **j**, **k** are in the body fixed coordinates of the rotating disc. The angles, ε, δ and γ are the Euler angles, used to define the angular position of the disc relative to the non-rotating coordinate system. A third coordinate system, defined by the basis vectors $\bar{\mathbf{i}}, \bar{\mathbf{j}}, \bar{\mathbf{k}}$, is also shown in the figure. The basis vectors $\bar{\mathbf{i}}, \bar{\mathbf{j}}$ always remain in the same plane as the disk (i.e. in the plane of vectors **i** and **j**) while vector $\bar{\mathbf{i}}$ also remains in the **i**-**I** plane. The position of the disc in Figure 3 can be described by three rotations about its body fixed axis. Initially the **i**, **j**, **k** and the **I**, **J**, **K** directions are coincident. The instantaneous position shown in Figure 3 can be obtained by rotating the disc, first about **j** than about **i** and **k** by the angles ε, δ and γ respectively.

If the disc is connected to a shaft that rotates about the unit vector $\mathbf{K}$ with an angular velocity $\dot{\theta}$, the angular velocity vector $\mathbf{\Omega}$ of the disc can be expressed as

$$\mathbf{\Omega} = \dot{\alpha}(t)\mathbf{i} + \dot{\beta}(t)\mathbf{j} + \dot{\theta}(t)\mathbf{K} \tag{6}$$

where, $\dot{\alpha}$ and $\dot{\beta}$ are the angular velocities of the disc relative to the rotating coordinate about unit vectors $\mathbf{i}$ and $\mathbf{j}$.

Expressing equation (6) in the $\bar{\mathbf{i}}, \bar{\mathbf{j}}, \bar{\mathbf{k}}$ basis using the coordinate transformations

$$\mathbf{i} = \text{Cos}\,\gamma\,\bar{\mathbf{i}} - \text{Sin}\,\gamma\,\bar{\mathbf{j}}$$
$$\mathbf{j} = \text{Sin}\,\gamma\,\bar{\mathbf{i}} + \text{Cos}\,\gamma\,\bar{\mathbf{j}}$$

and

$$\mathbf{K} = -\text{Sin}\,\varepsilon\,\bar{\mathbf{i}} + \text{Sin}\,\delta\,\text{Cos}\,\varepsilon\,\bar{\mathbf{j}} + \text{Cos}\,\delta\,\text{Cos}\,\varepsilon\,\bar{\mathbf{k}}$$

yields

$$\begin{aligned}\mathbf{\Omega} &= \left(\dot{\alpha}\text{Cos}\,\gamma - \dot{\beta}\text{Sin}\,\gamma - \dot{\theta}\,\text{Sin}\,\varepsilon\right)\bar{\mathbf{i}} \\ &\quad + \left(\dot{\alpha}\text{Sin}\,\gamma + \dot{\beta}\text{Cos}\,\gamma + \text{Sin}\,\delta\,\text{Cos}\,\varepsilon\right)\bar{\mathbf{j}} + \dot{\theta}\,\text{Cos}\,\delta\,\text{Cos}\,\varepsilon\,\bar{\mathbf{k}}\end{aligned} \tag{7}$$

On the other hand, the angular velocity $\mathbf{\Omega}$ of the disc is also

$$\mathbf{\Omega} = \dot{\delta}\bar{\mathbf{i}} + \dot{\varepsilon}\,\text{Cos}\,\delta\;\bar{\mathbf{j}} + \dot{\gamma}\,\bar{\mathbf{k}} \tag{8}$$

Equations (7) and (8) both express the angular velocity of the disc in the same coordinate system. Hence, their components must be equal. That is,

$$\begin{aligned}&\dot{\alpha}\text{Cos}\,\gamma - \dot{\beta}\text{Sin}\,\gamma - \dot{\theta}\,\text{Sin}\,\varepsilon = \dot{\delta} \\ &\dot{\alpha}\text{Sin}\,\gamma + \dot{\beta}\text{Cos}\,\gamma + \dot{\theta}\,\text{Sin}\,\delta\,\text{Cos}\,\varepsilon = \dot{\varepsilon}\,\text{Cos}\,\delta \\ &\dot{\theta}\,\text{Cos}\,\delta\,\text{Cos}\,\varepsilon = \dot{\gamma}\end{aligned} \tag{9}$$

If angles δ and ε are small, then the equations (9) can be simplified to give

$$\begin{aligned}&\dot{\alpha}\text{Cos}\,\gamma - \dot{\beta}\text{Sin}\,\gamma - \dot{\theta}\,\varepsilon = \dot{\delta} \\ &\dot{\alpha}\text{Sin}\,\gamma + \dot{\beta}\text{Cos}\,\gamma + \dot{\theta}\,\delta = \dot{\varepsilon} \\ &\dot{\theta} = \dot{\gamma}\end{aligned} \tag{10}$$

Integrating the third equation in equations (10) implies $\theta = \gamma$, since, θ is defined to be zero whenever γ is zero, the constant of integration is also zero. Hence, substituting θ for γ and integrating yields the angular motion of the disc in its body-fixed axis in terms of the angles δ and ε .

$$\begin{aligned}\alpha &= \quad \delta\,\text{Cos}\,\theta + \varepsilon\,\text{Sin}\,\theta + \alpha_o \\ \beta &= -\delta\,\text{Sin}\,\theta + \varepsilon\,\text{Cos}\,\theta + \beta_o\end{aligned} \tag{11}$$

where, constants of integration are zero since α and β must be zero whenever δ and ε are zero.

The actual measured distances in a typical setup are indicated by Z_{X} and Z_{Y} in Figure 3. Using the small angle approximation Z_{X} and Z_{Y} can be related to angles δ and ε at a given radius r from the center as

$$\begin{aligned}Z_{\text{X}} &= -r\,\text{Tan}\,\varepsilon \cong -r\,\varepsilon \\ Z_{\text{Y}} &= \;\; r\,\text{Tan}\,\delta\,/\text{Cos}\,\varepsilon \cong r\,\delta\end{aligned} \tag{12}$$

Substituting equations (12) into equations (11) and recognizing $-z_x = r\,\beta$ and $z_y = r\,\alpha$ are axial displacements relative to the rotating coordinate system, axial displacements of the disc in the rotating coordinates at radius r are

$$\begin{aligned}z_x &= -r\,\beta = \;\; Z_{\text{X}}\,\text{Cos}\,\theta + Z_{\text{Y}}\,\text{Sin}\,\theta \\ z_y &= \quad r\,\alpha = -Z_{\text{X}}\,\text{Sin}\,\theta + Z_{\text{Y}}\,\text{Cos}\,\theta\end{aligned} \tag{13}$$

Equations (13) are identical to equations (5) except that the direction of motion is axial rather than radial.

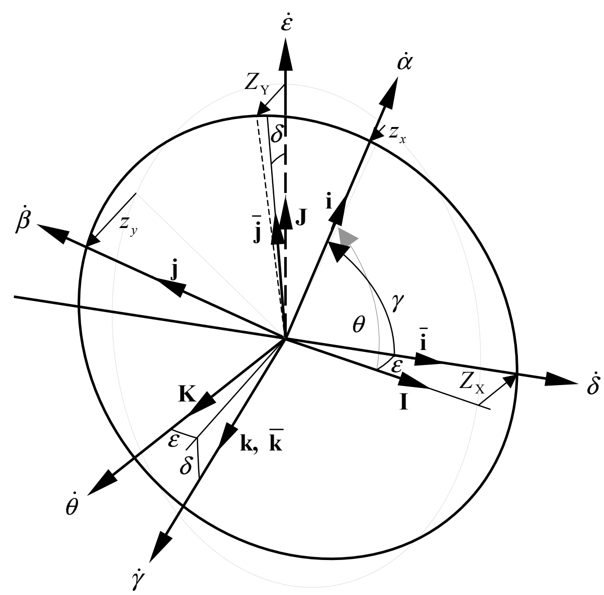


Figure 3. Definition of the stationary **I**, **J**, **K** and the rotating **i**, **j**, **k** basis, angular displacements and velocities.

Relationships in the Frequency Domain

Coordinate transformation equations (5) and (13) may both be expressed in frequency domain if the angular velocity of the rotating body is constant i.e. $\dot{\theta} = \omega_o = \text{Constant}$. The frequency domain coordinate transformation enables the use of spectral vibration data. Benefits of such procedure are less processing time, spectral averaging during acquisition and being able to make use of most data acquisition equipment, which are set to immediately post process time data to obtain their averaged spectra. However, it should be noted that the vibration data in auto spectrum format cannot be used since it does not contain any phase information.

Transforming equations (5) and (13) into their frequency domain counterparts requires the evaluation of the Fourier integral without explicit knowledge of the time functions $X(t)$, $Y(t)$, $Z_X(t)$ and $Z_Y(t)$. Here, only the transformation of equations (5) will be shown, since transformation of the other set is identical.

The essential problem is to evaluate the Fourier integral for the product of an implicit time function, say $f(t)$, with $\text{Cos}(\theta)$ and $\text{Sin}(\theta)$ functions. The function $f(t)$ represents functions $X(t)$ or $Y(t)$.

Hence, the integral to be evaluated is

$$\int_{-\infty}^{\infty} f(t) \begin{Bmatrix} \text{Cos}(\theta) \\ \text{Sin}(\theta) \end{Bmatrix} \text{e}^{-j\omega t} \, dt \ . \tag{14}$$

If the angular velocity of the rotating body is constant, i.e. $\theta = \omega_o t$, the Fourier integral given by (14) can be written in an alternative form using the exponential form of Cosine and Sine functions to obtain

$$\begin{aligned} &\frac{1}{2}\int_{-\infty}^{\infty} f(t)\,\text{e}^{-j(\omega-\omega_o)t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} f(t)\,\text{e}^{-j(\omega+\omega_o)t}\,dt \\ &\frac{1}{2j}\int_{-\infty}^{\infty} f(t)\,\text{e}^{-j(\omega-\omega_o)t}\,dt - \frac{1}{2j}\int_{-\infty}^{\infty} f(t)\,\text{e}^{-j(\omega+\omega_o)t}\,dt \end{aligned} \tag{15}$$

for the $\text{Cos}(\theta)$ and the $\text{Sin}(\theta)$ respectively.

Letting ($\omega - \omega_o = \psi_1$) and ($\omega + \omega_o = \psi_2$)the first integrals in (15) are Fourier transforms into ψ_1 domain where the spectrum of $f(t)$ is shifted towards higher frequencies by an amount equal to ω_o while the second integrals are transforms into ψ_2 domain where the spectrum of $f(t)$ is shifted towards lower frequencies by an amount equal to ω_o . This result is a well known property of the Fourier transforms theory.

Using the shifting property the frequency domain counterpart of equations (5) are written as

$$\begin{aligned} \underline{x}(\psi_1,\psi_2) &= \frac{1}{2}\left[\underline{X}(\psi_1) + \underline{X}(\psi_2)\right] - \frac{j}{2}\left[\underline{Y}(\psi_1) - \underline{Y}(\psi_2)\right] \\ \underline{y}(\psi_1,\psi_2) &= \frac{1}{2}\left[\underline{Y}(\psi_1) + \underline{Y}(\psi_2)\right] + \frac{j}{2}\left[\underline{X}(\psi_1) - \underline{X}(\psi_2)\right] \end{aligned} \tag{16}$$

where, the under bar indicates the complex spectrum of the associated function.

Equations (16) indicate that the coordinate transformation from stationary coordinates to rotating coordinates can be accomplished by shifting the measured vibration spectra toward lower and upper frequencies by an amount equal to the circular rotation frequency ω_o of the rotating body and summing the results appropriately.

Order Shift

A special case of frequency shifting due to transmission of periodic vibrations from rotating to stationary systems is known as the order shift. Order is a name given to the vibration frequencies of rotating shafts that are multiples of their rotation frequency, that is, the frequency of vibration equal to shaft's rotation frequency is called the first order, the vibration frequency at twice the shaft's frequency the second order and so on. When order related vibrations are transmitted to or measured in non-rotating coordinate systems the frequency shifts occur at an amount equal to one order and they may be towards higher and/or lower frequencies. The following derivation demonstrates how an order shift occurs.

Let a shaft rotate at a constant speed and the frequency of its rotation is *f*. Then the angular displacement of the shaft due to its rotation is

$$\theta = 2\pi f t \tag{17}$$

If the vibrations of a rotating shaft, in the rotating basis, are represented by periodic functions $x(t)$ and $y(t)$ with a fundamental frequency *f*, they can be expanded into Fourier series with a fundamental period $T = 1/f$.

$$\begin{aligned} x(t) &= \sum_{i=0}^{\infty} x_i \,\text{Sin}(2i\pi f t + \varphi_{xi}) \\ y(t) &= \sum_{i=0}^{\infty} y_i \,\text{Sin}(2i\pi f t + \varphi_{yi}) \end{aligned} \tag{18}$$

where, x_i and y_i are Fourier series coefficients, *f* is the rotation frequency, t is time, φ_{xi} and φ_{yi} are the phase of each series component. Substituting equations (17) and (18) into equations (4) yields,

$$X(t) = \sum_{i=0}^{\infty} x_i \operatorname{Sin}(2i\pi ft + \varphi_{xi}) \operatorname{Cos}(2\pi ft) - \sum_{i=0}^{\infty} y_i \operatorname{Sin}(2i\pi ft + \varphi_{yi}) \operatorname{Sin}(2\pi ft)$$

$$Y(t) = \sum_{i=0}^{\infty} x_i \operatorname{Sin}(2i\pi ft + \varphi_{xi}) \operatorname{Sin}(2\pi ft) + \sum_{i=0}^{\infty} y_i \operatorname{Sin}(2i\pi ft + \varphi_{yi}) \operatorname{Cos}(2\pi ft) \tag{19}$$

Equations (19) can be rearranged to yield

$$X(t) = \sum_{i=0}^{\infty} A_i \operatorname{Sin}[2(i+1)\pi ft + \mathbf{\Phi}_{Ai}] + \sum_{i=0}^{\infty} B_i \operatorname{Sin}[2(i-1)\pi ft + \mathbf{\Phi}_{Bi}]$$

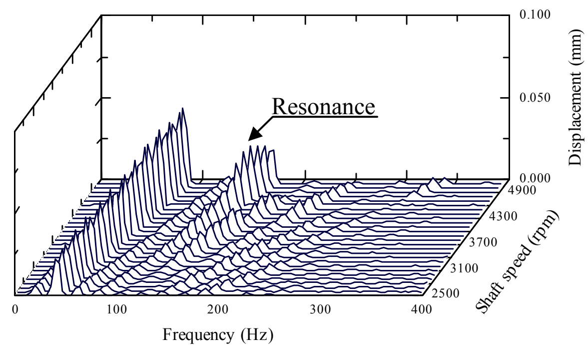
$$Y(t) = \sum_{i=0}^{\infty} C_i \operatorname{Sin}[2(i+1)\pi ft + \mathbf{\Phi}_{Ci}] + \sum_{i=0}^{\infty} D_i \operatorname{Sin}[2(i-1)\pi ft + \mathbf{\Phi}_{Di}] \tag{20}$$

where, Fourier coefficients A_i, B_i, C_i, D_i and the phase angles $\mathbf{\Phi}_{Ai}$, $\mathbf{\Phi}_{Bi}$, $\mathbf{\Phi}_{Ci}$, $\mathbf{\Phi}_{Di}$ are all functions of x_i, y_i, φ_{xi} and φ_{yi}. The Fourier transform of the series (20) will yield components at frequencies $(i+1)f$ and $(i-1)f$ in the non-rotating basis which correspond to the frequency if in the rotating basis. The shifting of the vibration frequencies to upper and lower frequencies by an amount equal to the rotation frequency of the shaft in this manner is known as the order shift. When a certain order in the rotating basis shifts due to transmission to the non-rotating basis, normally, two frequency components related to that order are observed. However, their amplitudes are not necessarily equal and depend on the amplitudes and the phase angles of the corresponding frequency components in the rotating basis. It is common to see an order shift in one direction only, when one or more of the coefficients A_i, B_i, C_i or D_i is zero.

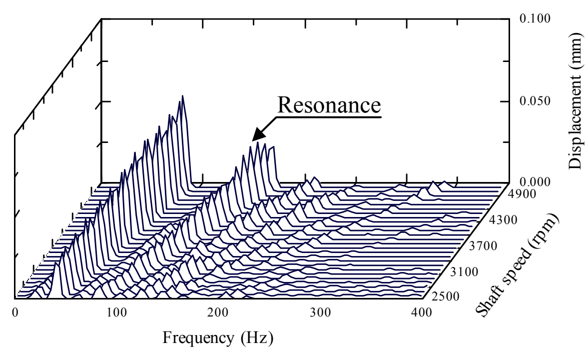
AN EXAMPLE

If the vibrations of a rotating shaft are measured in a non-rotating coordinate system, equations (5) or (16) may be applied to transform the data to the rotating coordinate system of the shaft. This transformation gives the true static bending and vibration frequencies of a rotating shaft.

In order to provide an example of such a case, bending vibrations of a V6 engine crankshaft were measured at flywheel using two non-contact displacement transducers as depicted in Figure 1, at a number of rotational speeds of the crankshaft. The spectral amplitudes of the measured displacements are plotted in Figure 4, in a waterfall plot format, where the familiar half and integer order vibrations are clearly visible.



(a)

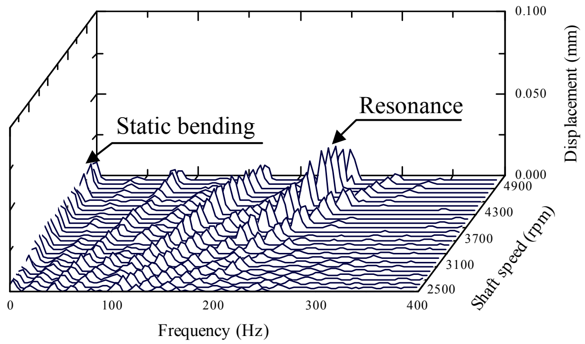


(b)

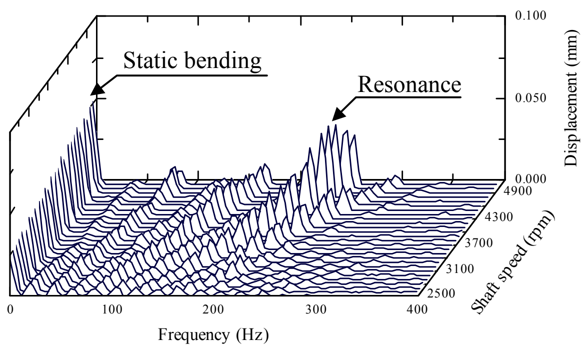
Figure 4. Vibration waterfall plots of a rotating crankshaft measured in a stationary coordinate system. (a) In stationary *X* direction. (b) In stationary *Y* direction.

If the plots of Figure 4 are interpreted without a coordinate transformation one may arrive at the following conclusions. The first order vibrations increase with the shaft speed and indicate bending by the inertial forces acting on the shaft. The amplitude of deflection is not the same as measured by each transducer. The X transducer measures a 0.047 mm while the Y transducer measures 0.056 mm displacement at the maximum shaft speed of 5100 rpm. The difference between the X and Y measurements is not known at this point. In fact, if a single transducer was used to obtain this measurement at either X or Y location the result would have been either 0.047 or 0.056 mm. In addition to static bending of the shaft, one would also conclude that a resonance of the shaft is excited by the second order forces at around 4800 rpm, hence, it must be about 160 Hz.

However, applying the coordinate transformation, equations (5) or (16), one would get a very different picture. The result of transformations is shown in Figure 5. Here, most significant vibrations are at zero and the third orders. The spectral



(a)



(b)

Figure 5. Vibration waterfall plots of a rotating crankshaft obtained by a coordinate transformation. (a) In rotating *x* direction. (b) In rotating *y* direction.

components at zero order indicate the static bending of the crankshaft. The bending is about 0.051 mm at 5100 rpm. This displacement is obtained by summing the squares of the static *x* and *y* deflections and taking the square root of the result. Also the direction of the static bending can be calculated. In this case the shaft is statically deflected at about 79 degrees from the rotating *x* axis. Since the rotating *x* axis is in the direction of #1 crank arm, the bending direction of the crank is exactly known. Furthermore, the crank resonance is excited by third order forces and its frequency is 240 Hz. This 80 Hz frequency difference from what the initial measurement indicated is very important especially if the shear force at the crank-flywheel interface is to be estimated from this data.

It is clear from the example above that when vibrations of rotating shafts or discs are measured from non-rotating coordinates, a coordinate transformation must be applied to determine the true bending amplitudes and frequencies.

Conclusions

This article explains the frequency shifts in the vibrations of rotating shafts and discs when such vibrations are observed from non-rotating coordinate systems. The phenomenon is simply the result of a coordinate transformation. Time and frequency domain formulations are given in the article for the transformation of measured data between the rotating and non-rotating coordinate systems. The time domain transformations are more general and apply even when the rotational speed of the body varies. The frequency domain formulation is numerically more efficient yet it only applies when rotational speed is constant. Another advantage of the frequency domain formulation is to be able to transform windowed data. A special case of frequency shifts, known as *order shift*, is also explained via the use of Fourier series expansions and a numerical example.

As explained in the example, the vibration spectra of a rotating shaft may be significantly different then what is observed from a non-rotating coordinate system. A behavior, often difficult to predict without a coordinate transformation. The transformation of vibration signals reveals the true static bending and vibration frequencies of a rotating shaft or disc, which, otherwise would only be possible through more elaborate and expensive test setups that use transducers connected to such bodies through slip rings.

The transformation equations given in the article are also suitable for transforming velocity measurements that typically employ LASER operated non-contact transducers. In such cases, however, the data must first be integrated to obtain displacements.

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